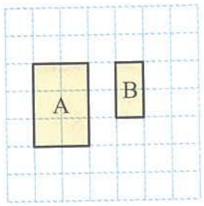
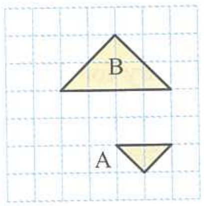
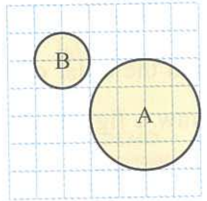
EXERCISE 16D

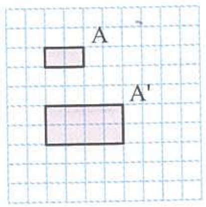
1 If A is the object and B is the image, state whether each of the following is an enlargement or a reduction:

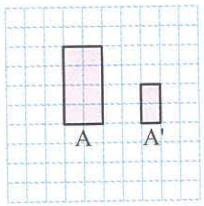
a


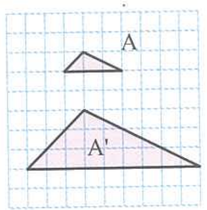
b


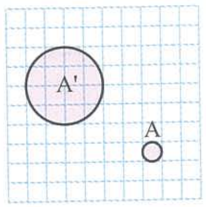
c


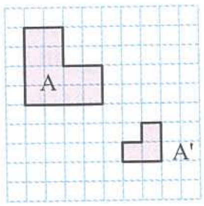
2 Find the scale factors for these enlargements and reductions:

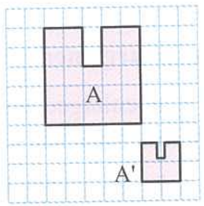
a


b


c


d


e


f


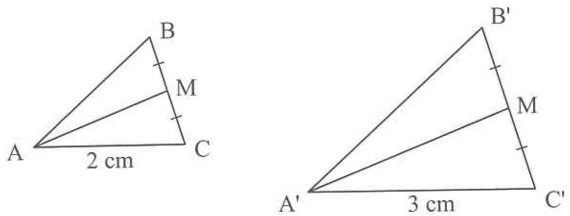
3 For each figure in **2**, what is the scale factor when A' is mapped back to A?

E SIMILAR FIGURES

Two figures are **similar** if one is an enlargement of the other.

Under an enlargement all distances between corresponding points are either increased or reduced by the scale factor k.

For example,



Since $k = \frac{3}{2}$,

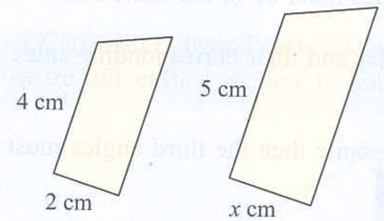
$A'M' = \frac{3}{2}(AM)$.

Notice that $\dfrac{A'B'}{AB} = \dfrac{B'C'}{BC} = \dfrac{A'C'}{AC} = \dfrac{A'M'}{AM} = \dfrac{3}{2}$.

Similar figures are **equiangular** and have corresponding **sides in the same ratio** or **same proportion.**

Example 6 — Self Tutor

Find x given these figures are similar.



Since the figures are similar, corresponding sides are in the same ratio.

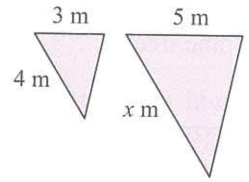
$\therefore \quad \dfrac{x}{5} = \dfrac{2}{4}$

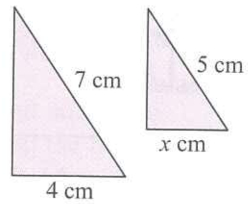
$\therefore \quad x = \frac{2}{4} \times 5$

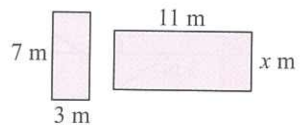
$\therefore \quad x = 2\frac{1}{2}$

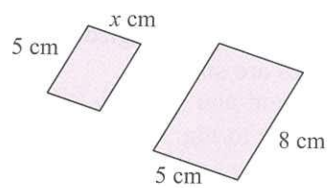
EXERCISE 16E

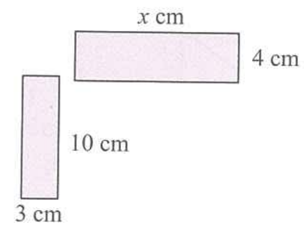
1 Find x given that the figures are similar:

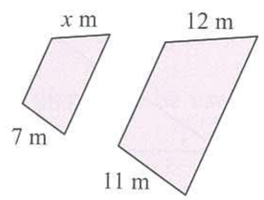
a


b


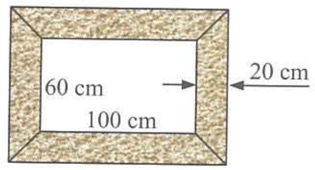
c


d


e


f


2 True or false? Give reasons for your answers.
- **a** All squares are similar.
- **b** All circles are similar.
- **c** All rectangles are similar.

3


A 20 cm wide picture frame surrounds a painting which is 100 cm by 60 cm.

Are the two rectangles shown here similar?

4 Sketch two quadrilaterals which:
- **a** are equiangular but not similar
- **b** have sides in proportion but are not similar.

F SIMILAR TRIANGLES

We have seen in **Exercise 16E** question **4** that quadrilaterals that are equiangular are not necessarily similar, and quadrilaterals that have sides in proportion are not necessarily similar.

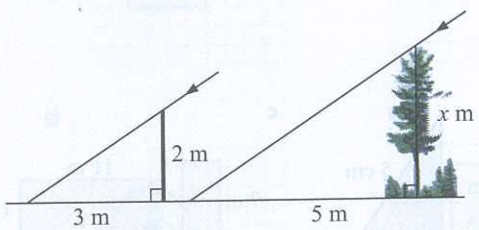
Triangles differ from quadrilaterals when it comes to similarity.

If triangles are equiangular then their corresponding sides must be in the same ratio.

> If two triangles are equiangular then they are similar and their corresponding sides are in the same ratio.

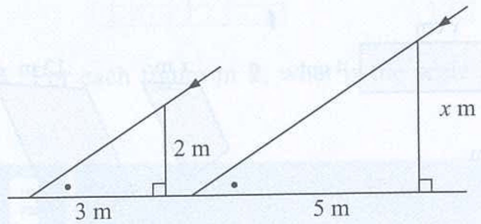
If we can show that two triangles have two angles the same then the third angles must also be equal and the triangles are similar.

Example 7 **Self Tutor**



At 4 pm one day the shadow of a pine tree was 5 m long. At the same time a 2 m long broom handle had a shadow which was 3 m long.

How high is the pine tree?



As the rays of light are parallel, we can mark the angles • {equal corresponding angles}.
Also each triangle is right angled.
$\therefore$ the triangles are similar.

So, $\frac{x}{5} = \frac{2}{3}$

$\therefore \quad x = \frac{2}{3} \times 5$

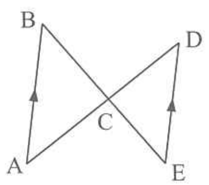
$\therefore \quad x = \frac{10}{3} \approx 3.33$

So, the pine tree is about 3.33 m high.

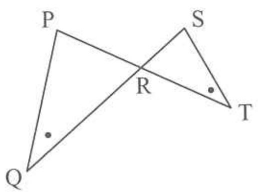
EXERCISE 16F

1 Give brief reasons why these figures possess similar triangles:

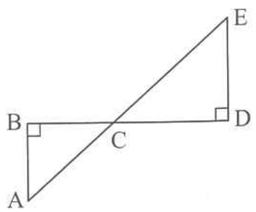
a



b

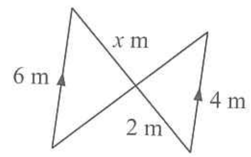


c

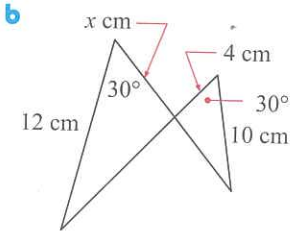


2 After establishing similarity, find the unknowns in:

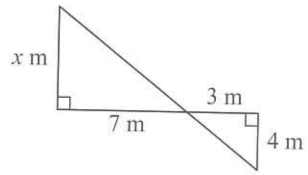
a



b

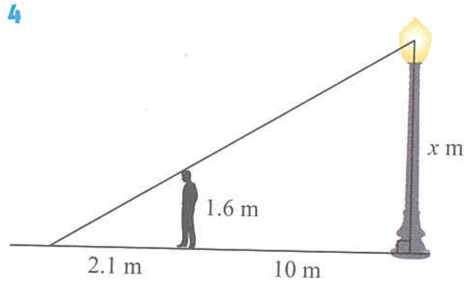


c



3 A Canadian redwood casts a shadow which is 9.4 m long at the same time as a vertical metre rule casts a shadow which is 1.32 m long. How high is the tree?

4



When a 1.6 m tall person stands 10 m from the base of an electric light pole the shadow of the person is 2.1 m long.

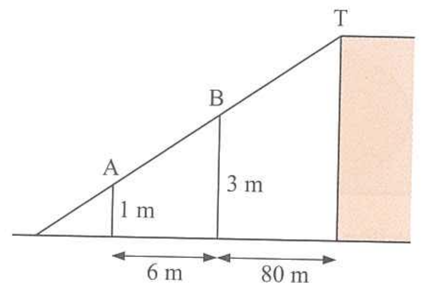
a Establish similarity of triangles.

b Find the height of the globe above ground level.

5 The diagram alongside has not been drawn to scale.

A building sits on horizontal ground and Yong uses two poles 6 m apart to help find the height of the building.

Pole A is 1 m high and pole B is 3 m high.



a By drawing one horizontal line explain how the information given can be used to find the height of the building.

b Find the height of the building to the nearest 10 cm.

G AREAS AND VOLUMES OF SIMILAR OBJECTS

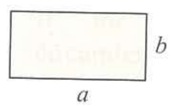
AREAS

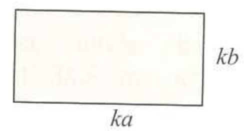
If the sides of a rectangle are multiplied by k, a similar rectangle is obtained.

The new area $= ka \times kb$

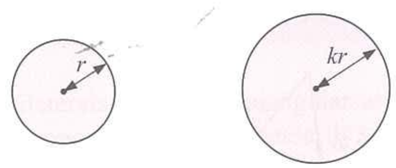
$= k^2ab$

$= k^2 \times$ the old area





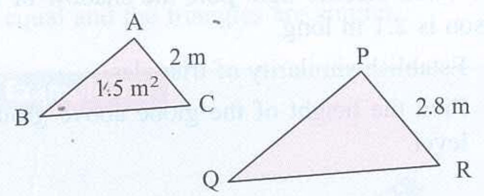
If the radius of a circle is multiplied by k, a similar circle is obtained.



The new area $= \pi \times (kr)^2$
$= \pi \times k^2 r^2$
$= k^2 \pi r^2$
$= k^2 \times$ the old area

If an object or figure is enlarged by a scale factor of k, then the area of the image $= k^2 \times$ the area of the object.

Example 8 — Self Tutor



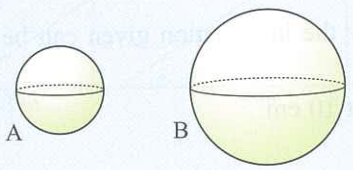
Triangles ABC and PQR are similar and the area of triangle ABC is 1.5 m^2.

What is the area of triangle PQR?

If we enlarge ΔABC to make ΔPQR, $k = \dfrac{2.8}{2} = 1.4$

Area $\Delta PQR = k^2 \times$ area ΔABC
$= 1.4^2 \times 1.5$
$= 2.94$ m^2

Example 9 — Self Tutor



radius 4 cm radius r cm

Two spheres have surface areas of 201 cm^2 and 452 cm^2 respectively.

Find the radius of the larger sphere.

If we enlarge A into B, area of B $= k^2 \times$ area of A

$\therefore$ $452 = k^2 \times 201$

$\therefore$ $\frac{452}{201} = k^2$

$\therefore$ $k = \sqrt{\frac{452}{201}}$ $\{k > 0\}$

$\therefore$ $k \approx 1.50$

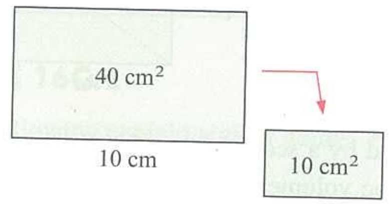
Now $r = k \times 4$

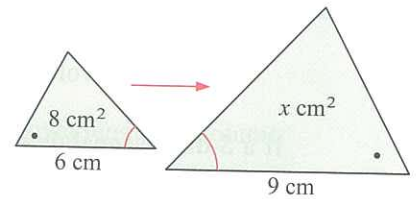
$\therefore$ $r \approx 6.00$

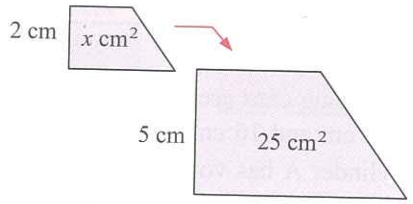
So, B has radius 6 cm.

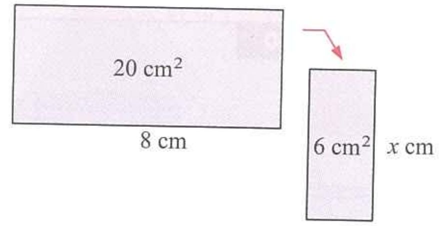
EXERCISE 16G.1

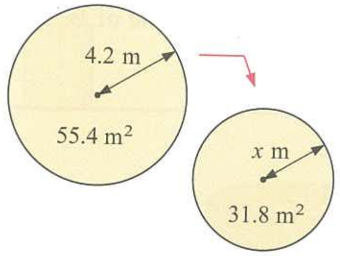
1. Consider the following similar shapes. Find:
   - i the scale factor
   - ii the length or area marked by the unknown.

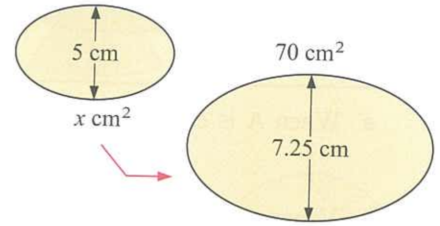
a


b


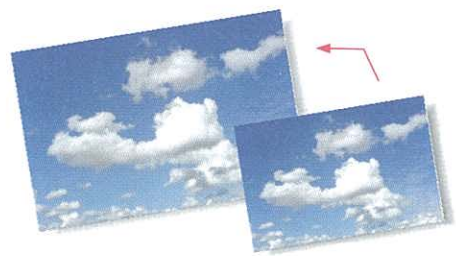
c


d


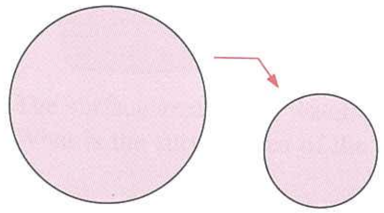
e


f


2. The sides of a triangular stained glass window pane are 5 m, 6 m and 7 m. Another pane is similar to it with longest side 21 m. Find:
   - a the scale factor
   - b the lengths of the other two sides
   - c the ratio of the areas of the two panes.

3. 

   Two rectangular photographs are similar and one has area double the other.
   - a What is the scale factor?
   - b If the larger one is 20 cm by 10 cm, what are the dimensions of the smaller one?

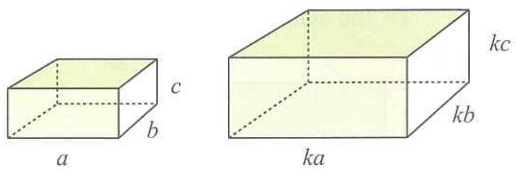
4. 

   Two circles are such that one has area three times the other.
   - a What is the scale factor?
   - b If the smaller circle has a circumference of 35.8 m, what is the circumference of the larger one?

VOLUMES

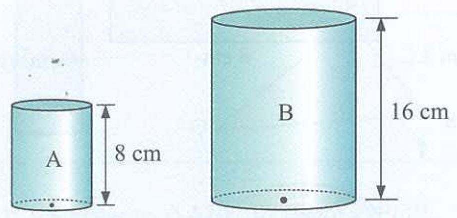
If the sides of a rectangular prism are multiplied by k, a similar prism is obtained.

The new volume $= ka \times kb \times kc$
$= k^3 abc$
$= k^3 \times$ old volume



If a 3-dimensional object is enlarged by a scale factor of k, then the volume of the image $= k^3 \times$ the volume of the object.

Example 10 — Self Tutor



Two soup cans are similar and have heights of 8 cm and 16 cm respectively. Cylinder A has volume 225 cm^3.

Find: **a** the ratio of their radii
b the volume of B.

a When A is enlarged to give B, $k = \frac{16}{8} = 2$

$\therefore$ the ratio of radii $= 1 : 2$

b Volume of B $= k^3 \times$ volume of A
$= 2^3 \times 225$ cm^3
$= 1800$ cm^3

Example 11 — Self Tutor

Two rectangular prisms have volumes of 48 m^3 and 6 m^3 respectively. The larger one has length 5 m. Find:

a the scale factor **b** the corresponding length of the smaller prism
c the ratio of their surface areas.

a Volume of larger $= k^3 \times$ volume of smaller
$\therefore \quad 48 = k^3 \times 6$
$\therefore \quad k^3 = 8$
$\therefore \quad k = 2$

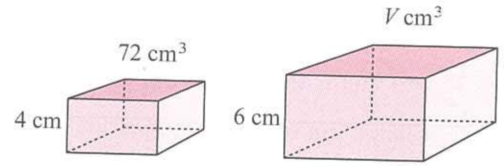
b Length of larger $= k \times$ length of smaller
$\therefore \quad 5 = 2 \times x$
$\therefore \quad \frac{5}{2} = x$
So, the smaller one has length 2.5 m.

c Area of larger $= k^2 \times$ area of smaller
$\therefore \quad \dfrac{\text{area of larger}}{\text{area of smaller}} = k^2 = 4$
$\therefore$ the ratio is $4 : 1$.

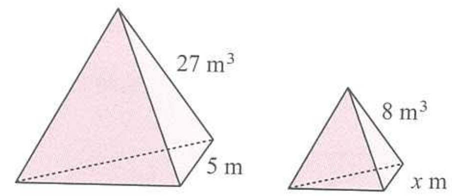
EXERCISE 16G.2

1 The following contain similar solids. Find the unknown length or volume:

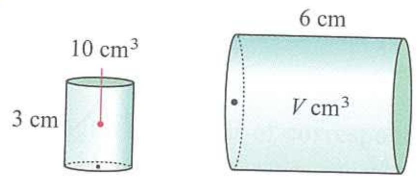
a



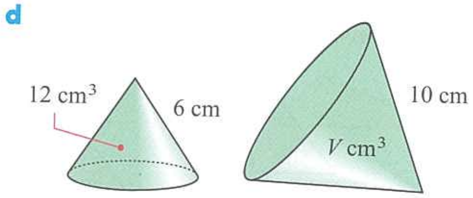
b



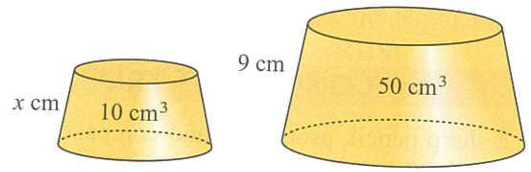
c



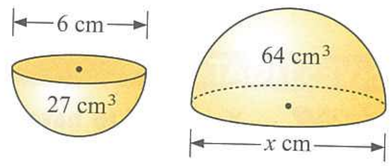
d



e



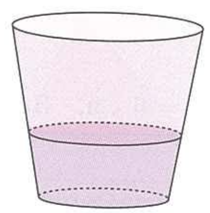
f



2 The surface areas of two similar cylinders are 6 cm^2 and 54 cm^2 respectively.
a Find the scale factor for the enlargement.
b If the larger cylinder has height 12 cm, how high is the smaller one?
c What is the ratio of their volumes?

3 Two similar cones have volumes of 4 cm^3 and 108 cm^3 respectively.
If the larger one has surface area 54 cm^2, find the surface area of the smaller one.

4



Two buckets are similar in shape. The smaller one is 30 cm tall and the larger one is 45 cm tall. Both have water in them to a depth equal to half of their heights. The volume of water in the smaller bucket is 4400 cm^3.

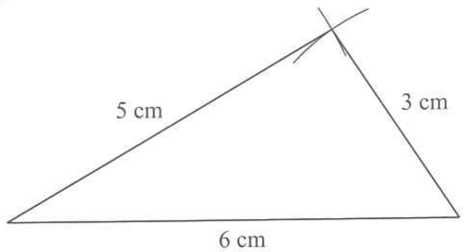
a What is the scale factor in comparing the bucket sizes?
b What is the volume of water in the larger bucket?
c The surface area of the water in the larger bucket is 630 cm^2.
What is the surface area of the water in the smaller bucket?

H CONGRUENCE OF TRIANGLES

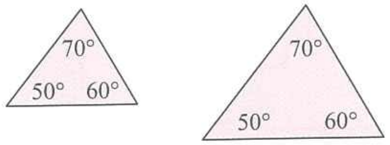
Two triangles are **congruent** if they are identical in every respect apart from position.

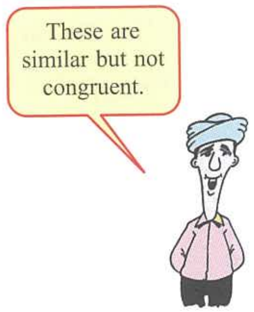
If we are given the lengths of three sides of a triangle we can construct it in only one way.

For example, the triangle with sides of length 3 cm, 5 cm and 6 cm is shown opposite.



If we are given the measures of three angles of a triangle we can construct it in more than one way.





What other information do we need to know that would allow us to construct a triangle in only one way?

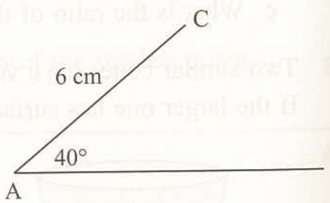
INVESTIGATION — CONGRUENT TRIANGLES

You need to have: a ruler, a sharp pencil, protractor, and compass.

What to do:

1 Accurately construct triangle ABC with $AB = 6$ cm, $B\widehat{A}C = 45^o$ and $AC = 4$ cm. Is there one and only one possible triangle?

2 Draw a horizontal line with A at the left end.
Use your protractor to measure an angle of 40^o at A.
Draw [AC] with length 6 cm.
With compass point on C draw an arc of a circle with radius 5 cm to cut the base line at B.



- How many possible positions are there for B?
- How many triangles ABC can be drawn given $AC = 6$ cm, $B\widehat{A}C = 40^o$ and $BC = 5$ cm?

3 Now draw a horizontal line with B at the right end. At B draw a right angle and on the perpendicular mark C where $BC = 3$ cm. With C as centre, draw an arc of radius 4 cm to cut the original base line at A.
How many different triangles can be drawn from this information?

4 Accurately draw triangle ABC in which $A\widehat{B}C = 40^o$, $A\widehat{C}B = 50^o$ and $BC = 4$ cm. How many different triangles can be drawn from this information?

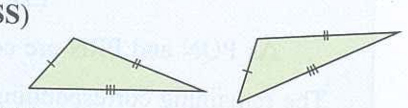
5 Accurately draw triangle ABC in which $A\widehat{B}C = 40^o$, $A\widehat{C}B = 50^o$ and a side other than [BC] has length 4 cm.
How many different triangles can be formed now?

6 Write a summary indicating when one and only one triangle can be drawn from given information involving 3 angles or sides.

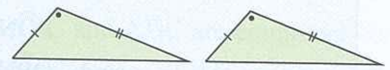
If a given set of information allows us to construct a triangle in only one way, then this information is sufficient to state that two triangles are congruent. From the investigation above, therefore, we can state four acceptable tests for congruence:

Two triangles are **congruent** if any one of the following is true:

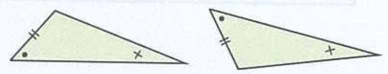
- All corresponding sides are equal in length. **(SSS)**



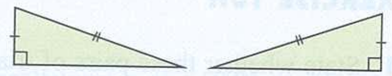
- Two sides and the **included** angle are equal. **(SAS)**



- Two angles and a pair of **corresponding sides** are equal. **(AAcorS)**



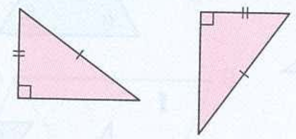
- For right angled triangles, the hypotenuses and one pair of sides are equal. **(RHS)**



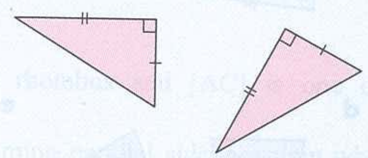
Example 12 — Self Tutor

Are these pairs of triangles congruent? Give reasons for your answers.

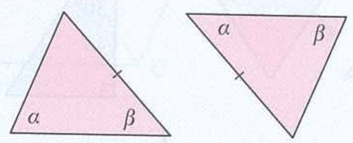
a



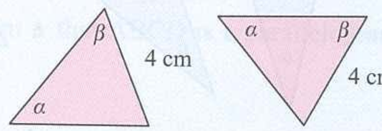
b



c



d

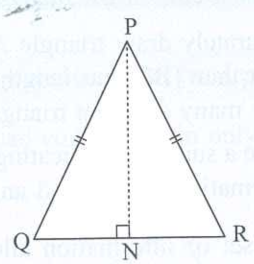


a Yes {RHS}
b Yes {SAS}
c No. This is **not** AAcorS as the equal sides are not opposite the same sized angle.
d Yes {AAcorS}

Example 13 — Self Tutor

Are there congruent triangles in the given figure?

If so, what can be deduced?



In Δs PQN and PRN: (1) PQ = PR {given}

(2) $P\widehat{N}Q = P\widehat{N}R = 90^o$ {given}

(3) [PN] is common to both

$\therefore$ Δs PQN and PRN are congruent. {RHS}

The remaining corresponding sides and angles are equal, and so

QN = RN,

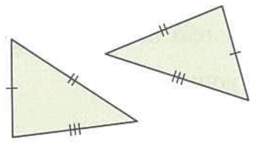
$P\widehat{Q}N = P\widehat{R}N$,

and $Q\widehat{P}N = R\widehat{P}N$

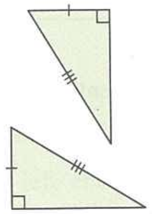
EXERCISE 16H

1 State whether these pairs of triangles are congruent, giving reasons for your answers:

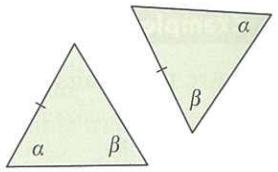
a



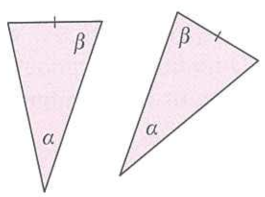
b



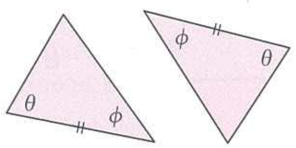
c



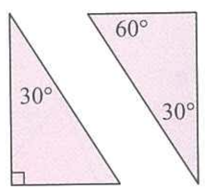
d



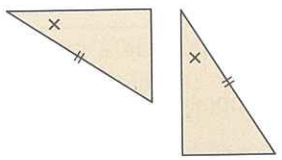
e



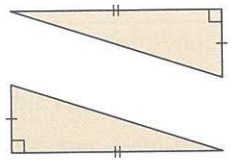
f



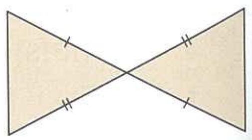
g



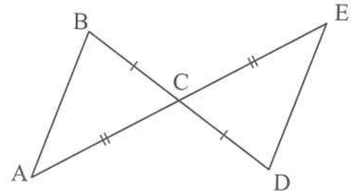
h



i



2



For the given figure, copy and complete:

In Δs CAB and CED

(1) CB = CD {........}

(2) AC = ... {........}

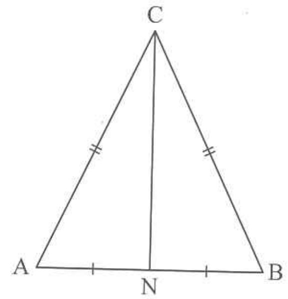
(3) .... = ... {..................}

$\therefore$ these triangles are congruent. {........}

Consequently $A\widehat{B}C$ = ...

$\therefore$ [AB] $\parallel$ .... {equal ....}

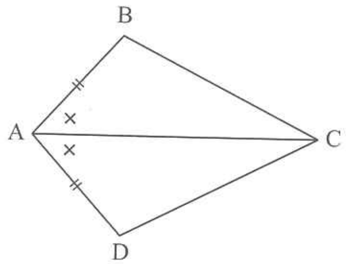
3



a Show that Δs CAN and CBN are congruent.

b What are the consequences of this congruence?

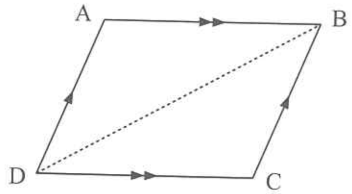
4



a Show that Δs ABC and ADC are congruent.

b What are the consequences of this congruence?

5

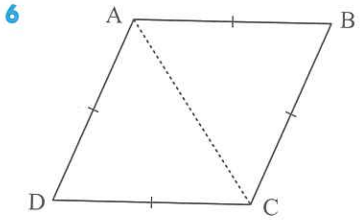


a ABCD is a parallelogram and [DB] is one of its diagonals.

Use properties of parallel lines to show that Δs ABD and CDB are congruent.

b What are the consequences of this congruence?

6



a ABCD is a rhombus and [AC] is one of its diagonals.

Without assuming parallel sides, explain why Δs ABC and ADC are congruent.

b Deduce from **a** that ABCD is a parallelogram.

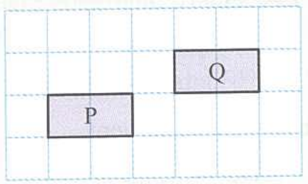
LINKS click here

THE MATHEMATICS OF AIR HOCKEY

Areas of interaction:
Approaches to learning

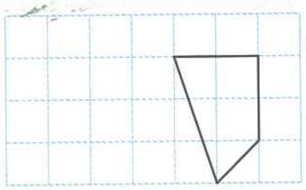
REVIEW SET 16A

1 **a**



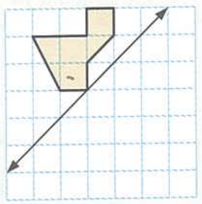
What translation vector moves Q to P?

b



On grid paper redraw the given figure and translate it $\begin{pmatrix} -4 \\ 1 \end{pmatrix}$.

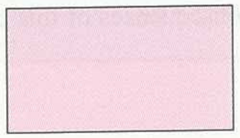
2



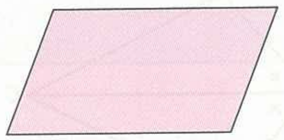
Copy and reflect the shape in the mirror line.

3 Draw lines of symmetry for:

a

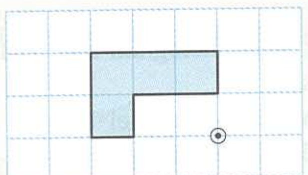


b

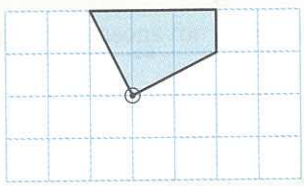


4 Copy and rotate the following figures about O clockwise through 90^o:

a

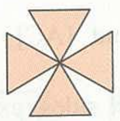


b

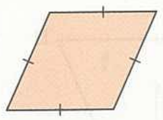


5 Find any points of rotational symmetry and state the order of rotational symmetry:

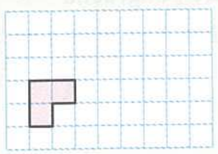
a



b



6



Copy and enlarge the shape with a scale factor $k = \frac{5}{2}$.

7

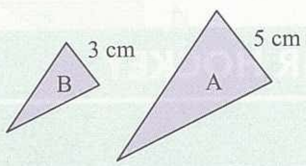
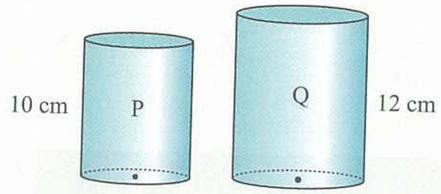


Figure A is reduced to figure B.

a What is the scale factor?

b If A has area 15 cm^2, find the area of B.

8



P and Q are similar solid cylindrical cans with heights as shown.

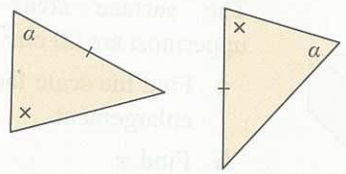
a What is the scale factor?

b If P has an end surface area of 50 cm^2, what is the end surface area of Q?

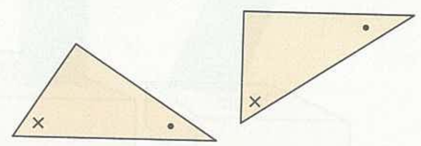
c If Q has volume 2400 cm^3, find the volume of P.

9 State whether these pairs of triangles are congruent, giving reasons for your answer:

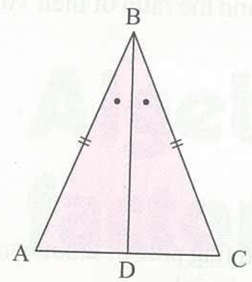
a



b



10



Triangle ABC is isosceles with BA = BC.

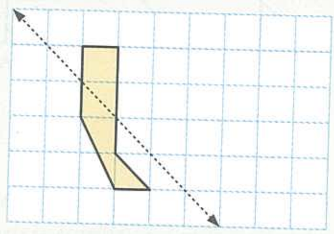
[BD] is the angle bisector of $A\hat{B}C$.

a Show that triangles DAB and DCB are congruent.

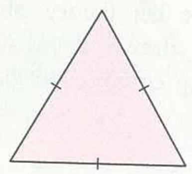
b Show that a consequence of **a** is that [BD] is perpendicular to [AC].

REVIEW SET 16B

1 Copy and reflect the shape in the mirror line:



2



a How many lines of symmetry does an equilateral triangle have?

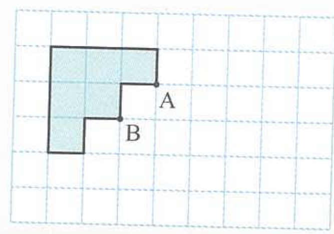
b Does it have a point of rotational symmetry?

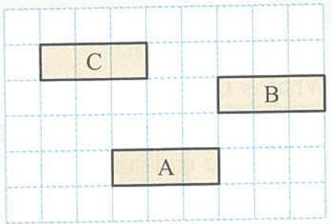
c Illustrate **a** and **b** on a sketch.

3 Copy and rotate this figure:

a clockwise about A through 180^o

b anticlockwise about B through 90^o.

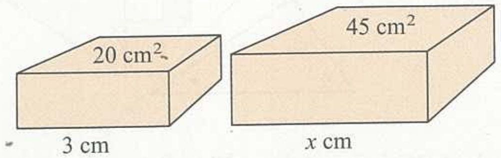


4 

What translation vector would move:

a C to B **b** B to A?

5 Draw a figure which possesses rotational symmetry of order 6.

6 

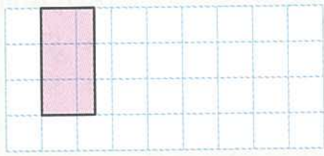
Two blocks of cheese are similar. The surface areas which are uppermost are 20 cm^2 and 45 cm^2.

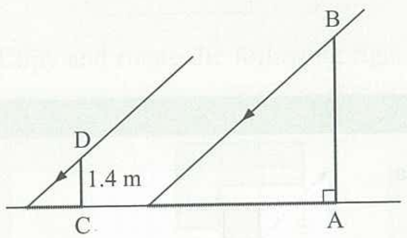
a Find the scale factor k for the enlargement.

b Find x.

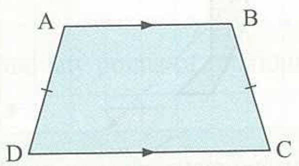
c Find the ratio of their volumes.

7 Copy the given figure and reduce it by a factor of $k = \frac{2}{3}$.



8 

[AB] is a vertical flag pole of unknown height. [CD] is a vertical stick. When the shadow of the flag pole is 12.3 m long, the shadow of the stick is 1.65 m long. Find, correct to 3 significant figures, the height of the flag pole.

9 

The figure alongside is called an *isosceles trapezium* as its non-parallel sides are equal in length.
Claudia is convinced that the angles at C and D are also equal. To prove her theory, she added two lines to the figure, drawn from A and B respectively. Explain using congruence the rest of Claudia's proof.

Chapter 17

Algebraic factorisation

Contents: